B.sc Part II (Honrs)
paper IV

Differential equation of the first order and first degree.

Variables separable: — If the given equation $Mdx + Ndy = 0$ Where M and N are functions of x and y or arbitrary Constants (but not of derivatives) can be put in the form $f(x)dx + \phi(y)dy = 0$ Where $f(x)$ is a function of x only and $\phi(y)$ is a function of y only, then we say that the variables are separable. The solutions of such equations can be easily found by direct integration, such that its solution is

$$\int f(x)dx + \int \phi(y)dy = k$$

Where k is an arbitrary Constant.

Ex:— $\frac{dy}{dx} = e^{x+y} + x^2 e^y$.

Solution:— The given equation can be written as

$$\frac{dy}{dx} = e^x \cdot e^y + x^2 e^y$$

$$= (e^x + x^2) e^y.$$

$$\Rightarrow \frac{dy}{e^y} = (e^x + x^2)dx$$

$$\Rightarrow e^{-y}dy = (e^x + x^2)dx$$

hence integrating, we get

$$\int e^{-y}dy = \int (e^x + x^2)dx$$

$$\Rightarrow \frac{e^{-y}}{-1} = e^x + \frac{x^3}{3} + c$$ Where c is a Constant.

$$\Rightarrow e^x + \frac{x^3}{3} - e^{-y} = k$$

Which is the required Solution.

Ex:- Solve $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$.

Solution:- Put $y = vx$, so that

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Therefore the given equation becomes

$$v + x\frac{dv}{dx} = v + \tan v$$

$$\Rightarrow x\frac{dv}{dx} = \tan v = \frac{dx}{x} = \cot v\, dv.$$

Integrating, we get

$$\int \frac{dx}{x} = \int \cot v\, dv$$

$$\Rightarrow \log x = \log \sin v + k$$

$= \log \sin v + \log c$, on writing $k = \log c$

$= \log c \sin v$

$\Rightarrow x = c \sin v = c \sin(y/x)$, which is the required solution.

Ex:- Solve $y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$.

Solution:- From the given equation, we get

$$y - x\frac{dy}{dx} = ay^2 + a\frac{dy}{dx} \Rightarrow x\frac{dy}{dx} + a\frac{dy}{dx} = y - ay^2$$

$$\Rightarrow (x+a)\frac{dy}{dx} = y(1-ay) \Rightarrow \frac{dx}{x+a} = dy/y(1-ay)$$

integrating we get

$$\int\frac{dx}{x+a} = \int\frac{dy}{y(1-ay)} = \int\left(\frac{1}{y} + \frac{a}{1-ay}\right)dy$$

$$\Rightarrow \log(a+x) = \log y + a\log(1-ay) \times \frac{1}{-a} + k$$

$$= \log y - \log(1-ay) + \log c \;(\because k = \log c) = \log(cy) - \log(1-ay) = \log\frac{cy}{1-ay}$$

$$\Rightarrow x+a = cy/1-ay \Rightarrow (x+a)(1-ay) = cy.$$

Which is the required Solution.

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